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Optical symmetry in liquid crystal displays

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Some authors include in their articles polar plots showing the directional dependence of either transmission or contrast ratio of a liquid crystal display (LCD). In some cases those plots have a symmetry axis; here we explain when and why it occurs.

1. Introduction

If we look at a polar plot giving the angular dependence of the contrast or transmission for a 90° twisted nematic display with crossed polarizers, at each side parallel with the rubbing direction of the alignment layers, it is clear that this plot has a symmetry axis (see figure 1). This axis lays at 45°, i.e. between the two polarizer directions and in the same direction as the director in the middle of the nematic layer. Similar conclusions can be made for supertwist displays and in general for all chiral nematic type displays [1, p. 4.24 . . .]. Before explaining the precise conditions for such symmetry in liquid crystal displays, let us first explain the underlying reasons.

2. General case

Consider an infinite two dimensional inhomogeneous plate of thickness l surrounded by a homogeneous dielectric. The plate lays parallel with the xy plane and between $z = -l/2$ and $z = l/2$; see figure 2. Suppose all of the material properties of this plate are symmetric for a 180° rotation around the axis $\mathcal{S}$ where $\mathcal{S}$ is an axis laying in the plane $z = 0$. We define the angle between the x axis and the $\mathcal{S}$ axis as ϑ_s.

Now consider a plane light wave $\mathbf{k}$ incident on the plate from underneath with incidence angle α_1 and azimuth angle β_1 as indicated in figure 2. The angles α and β are refered to the (x, y, z) axes. The wavevector $\mathbf{k}'$ of the transmitted wave is parallel to $\mathbf{k}$, so its orientation is also given by (α_1, β_1). Suppose we have a transmission $\mathcal{T}$ for this direction of light. Following the Helmholtz reciprocity theorem [2, p. 84] we must have the same transmission $\mathcal{T}$ if we reverse the travelling direction of the light wave. So the light wave $\mathbf{k}'' = -\mathbf{k}'$ entering from above also has transmission $\mathcal{T}$. Referred to the $(x, -y, -z)$ axes this light direction has angles $(\alpha_1, \pi - \beta_1)$. If we now rotate this situation by 180° around the $\mathcal{S}$ axis we still have the same plate due to the supposed symmetry of the dielectric medium, but a light wave entering from underneath with angles $(\alpha_1, \pi + 2\vartheta_s - \beta_1)$.

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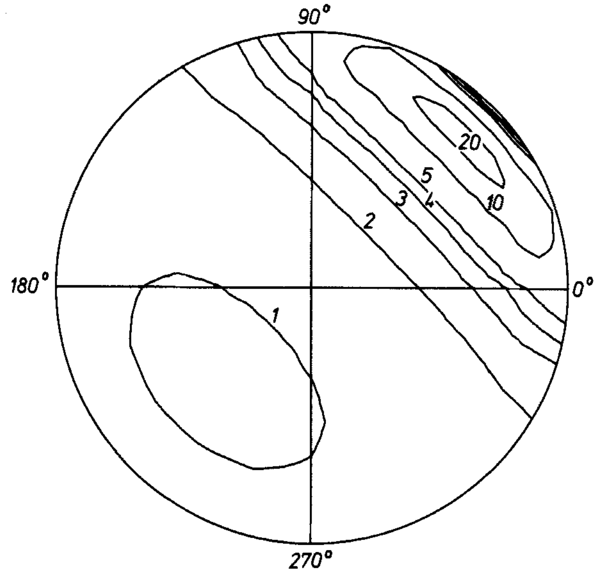

Figure 1. Calculated contrast for a 90° twisted nematic display with crossed polarizers.

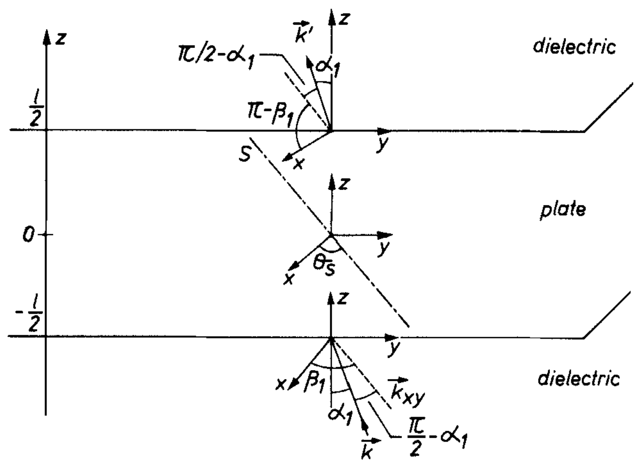

Figure 2. Definition of the axes.

We conclude that light waves (α_l, β_l) and (α_l, $\pi + 2\vartheta_s - \beta_l$) have the same transmission. Set out in a polar plot this causes the axis satisfying $\beta_l = \pi + 2\vartheta_s - \beta_l$ or $\beta_l = \vartheta_s + \pi/2$, i.e. the axis *perpendicular to* $\mathscr{S}$ to be the symmetry axis of the polar plot.

3. Application to chiral nematic type liquid crystal displays

Let us now examine how to use the theory of §2 in liquid crystal displays.

3.1. *The liquid crystal layer*

First we consider the liquid crystal layer. By rotating 180° around $\mathscr{S}$, the tilt $\varphi(z)$ and twist $\vartheta(z)$ distribution of the director, see figures 3 and 4 (figures 4 and 5 are only

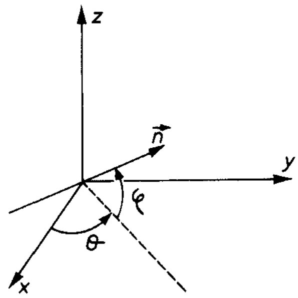

Figure 3. Definition of the tilt φ and the twist ϑ.

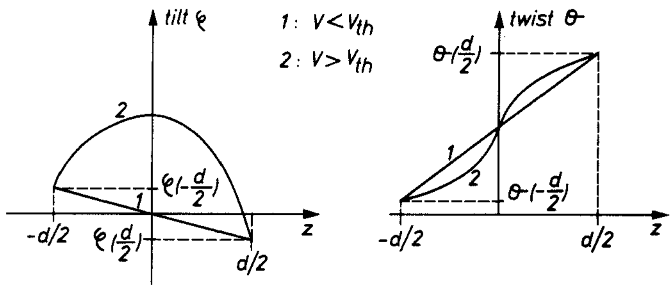

Figure 4. Tilt and twist with antisymmetric pretilt. Only one of the reverse tilt situations is shown.

sketches), will transform to

$$\left.\begin{aligned} \varphi'(z) &= -\varphi(-z), \\ \vartheta'(z) &= 2\vartheta_s - \vartheta(-z). \end{aligned}\right\} \tag{1}$$

Now, in order for the symmetry explained in §2 to be applicable, we require

$$\left.\begin{aligned} \varphi'(z) &= \varphi(z), \\ \vartheta'(z) &= \vartheta(z). \end{aligned}\right\} \tag{2}$$

Equations (1) and (2) can only be satisfied simultaneously if

$$\left.\begin{aligned} \varphi(-z) &= -\varphi(z), \\ \vartheta(-z) &= 2\vartheta_s - \vartheta(z). \end{aligned}\right\} \tag{3}$$

The reader should verify that this situation can only occur in a liquid crystal layer with antisymmetric pretilt condition: $\varphi(-d/2) = -\varphi(d/2)$ with d the total thickness of the liquid crystal layer. The symmetry axis of the liquid crystal layer is defined by

$$\vartheta_s = \frac{\vartheta(-d/2) + \vartheta(d/2)}{2}. \tag{4}$$

However as can be seen in figure 4 the symmetry is broken when a voltage above threshold is applied; the tilt is redistributed either clockwise or anti-clockwise (cf. the phenomena of reverse tilt [3]). Neither solution preserves the symmetry. Therefore neither can explain the symmetry seen at voltages above threshold; remember that a contrast diagram is merely the division of two transmission diagrams.

Still there is another way to satisfy the symmetry conditions of §2. Indeed, since the directors **n** and $-\mathbf{n}$ describe the same situation, the orientation $(\varphi(z), \vartheta(z))$ also can be written as

$$\left.\begin{aligned} \varphi''(z) &= -\varphi(z), \\ \vartheta''(z) &= \vartheta(z) + \pi. \end{aligned}\right\} \tag{5}$$

So we can also realize the symmetry discussed in §2 by satisfying

$$\left.\begin{aligned} \varphi'(z) &= -\varphi(z), \\ \vartheta'(z) &= \vartheta(z) + \pi. \end{aligned}\right\} \tag{6}$$

Combining equations (1) and (6) leads us to the condition

$$\left.\begin{aligned} \varphi(-z) &= \varphi(z), \\ \vartheta(-z) &= 2\vartheta_s - \pi - \vartheta(z). \end{aligned}\right\} \tag{7}$$

The reader should verify that this situation will occur with the symmetric pretilt condition $\varphi(-d/2) = \varphi(d/2)$. The symmetry axis in this case is defined by

$$\vartheta_s = \frac{\vartheta(-d/2) + \vartheta(d/2)}{2} + \pi/2. \tag{8}$$

It is clear from figure 5 that the present symmetry does not disappear at voltages above threshold. It is the latter symmetry that is normally seen.

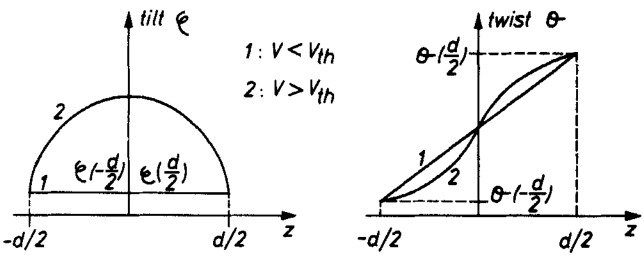

Figure 5. Tilt and twist with symmetric pretilt.

3.2. *Further display requirements*

Now consider a typical display structure constructed with a liquid crystal layer between two conducting glass plates and a polarizer at each side. To maintain the symmetry conditions, the glass plates as well as the polarizers must be identical. Moreover the polarizing directions must lay symmetrically with respect to $\mathscr{S}$. One

Summary of possible symmetries; $\xi = [\vartheta(-d/2) + \vartheta(d/2)]/2$, $\eta = [\vartheta(-d/2) + \vartheta(d/2)]/2 + \pi/2$.

Pretilt	Transmission		Contrast
	$V < V_{th}$	$V > V_{th}$	
$\varphi(-d/2) = \varphi(d/2) = 0$	ξ and η	η	η
$\varphi(-d/2) = -\varphi(d/2) \neq 0$	ξ	None	None
$\varphi(-d/2) = \varphi(d/2) \neq 0$	η	η	η
Otherwise	None	None	None

correct situation is when each polarizer direction is parallel with the rubbing direction of the alignment layer at its side. This is true for all values of the total twist. In the table we find which symmetry axes appear in all possible cases. Note that this table gives the symmetry axes of the polar plots, which are perpendicular to the symmetry axes of the display structure given in equations (4) and (8).

4. Examples

Figure 1 showed the contrast plot for a 90° TN display with perpendicular polarizers and 64 line multiplexing. Figure 6 shows the same display with parallel polarizers. Both plots are calculated for a display consisting of an 8 μm Merck ZLI 3244 liquid crystal layer doped with 0·05 wt % S811 using the software package GLUE (this is an in-house development of the Laboratory of Electronics). Liquid crystal orientations are calculated with a finite element method [4], the optical calculations are based on the 4 × 4 matrix method [5]. The total twist is 90° and the pretilt is zero; the glass plates are 1·1 mm thick and have a refractive index of 1·54. The polarizers are of type Polaroid HN42. For the calculations a 550 nm light source with 100 μm coherence length was assumed.

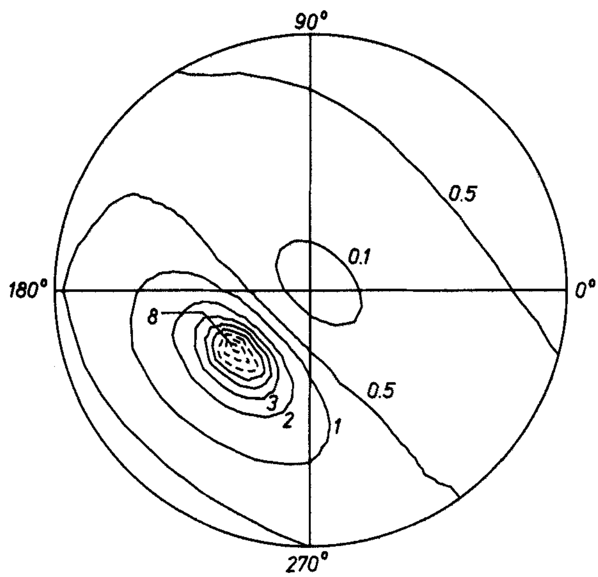

Figure 6. Polar plot of the contrast for a 90° twisted nematic display with parallel polarizers.

Figure 7 shows for the same display with crossed polarizers the transmission for a subthreshold regime. Since zero pretilt conforms with both type of symmetries mentioned in §3.1, we see two symmetry axes in this figure. Note that the maximum angle of incidence was 40° in figures 1, 6 and 7.

5. Conclusion

We have shown that polar plots of chiral nematic type displays can have symmetry axes and explained the symmetry conditions for the display to produce them. The results are summarized in the table. Only in the case of zero pretilt are there two

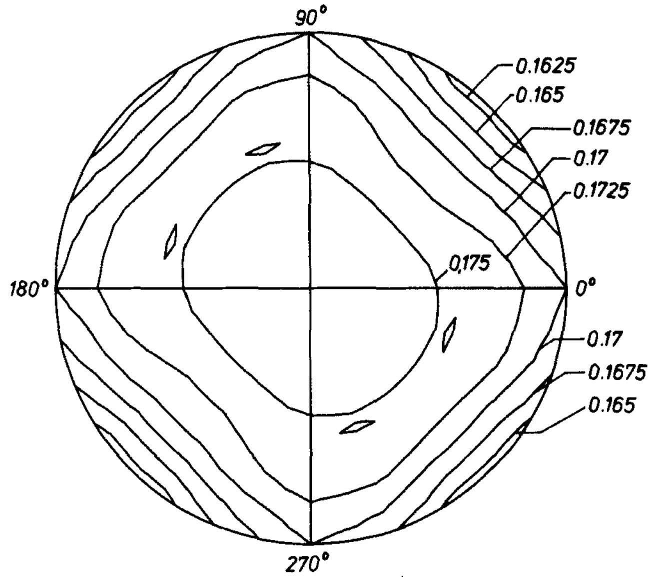

Figure 7. Polar plot of transmission for a 90° twisted nematic display with perpendicular polarizers.

symmetry axes; this is obvious since zero pretilt satisfies both cases discussed in §3.1. The case of symmetric non-zero pretilt is the one usually encountered in twisted nematic devices since it eliminates the effect of reverse tilt. It leads to a single symmetry axis.

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